Calculating vol skew change risk (skew-vega)

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Introduction

An interesting and important risk in an options portfolio is the impact of a changing implied volatility skew. It is not uncommon for implied volatility curves to become steeper or flatter on one side or both due to change in market bias, fear, expectation etc. This can also be driven by large trades on out of the money options, e.g. large risk reversals. It is not uncommon to see something like put buying and call selling on index options by protection seekers, which results in a change the volatility surface skew while possibly having little to no affect on the ATM vol. Thus the measure of the risk due to change in the skew is important information for a trader.

Defining a change in vol skew

There is no standard accepted definition for a change in vol skew. However in general it is a change in volatility of some OTM put(call) vol with the ATM vol remaining the same which could (but not necessarily) be coupled with a corresponding opposite move in the equivalent OTM call(put) vol.

e.g. + 1 vol move in the 30delta Puts would be example of just a changing put skew

or +1 vol move in 25d put with -1 vol move in 25d call

or 3% (relative) move in 25d put with a -3% move in 25d call   ……. (1)

We won't belabor over the definition and leave that as a trader's choice. However for the purpose of this paper, we will consider the change in skew to be defined as (1) above
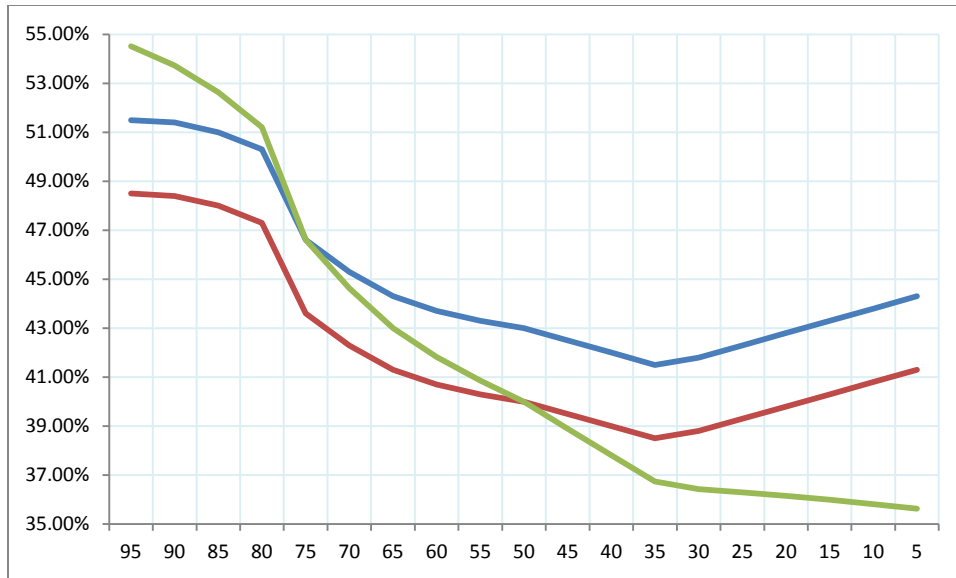
It is important to note that when a change in vol skew occurs, there is a defined change in vol for one given point, with the ATM remaining unchanged. Thus the vol surface is not moving parallel but each point on the vol surface is moving my some different amount based on the vol surface curve.   In regular vega calculations, all points move up or down by the same amount, but in Skew-vega this is not true.

e.g.

In the graph below, the Red line is the current vol surface for a given stock and given maturity.

A parallel shift in vol, which will be use to calculated normal vega is shown in the Blue line.

But for Skew-vega, the shifted vol surface may look something like the Green line.

55.00%
53.00%
51.00%
49.00%
47.00%
45.00%
43.00%
41.00%
39.00%
37.00%
35.00%

95 90 85 80 75 70 65 60 55 50 45 40 35 30 25 20 15 10 5

Effectively the vol skew change is a rotation of the curve around the ATM vol as the pivot point.

Issues with calculating Skew-vega

Calculating regular vega for a portfolio of options on a given stock is rather easy – simply add the vega across all options as the vol shift is parallel.

But in the case of Skew-vega, as described above, the vol shift for each option is different. Thus in order to calculate skew-vega, we need figure out the vol shift applicable to *each* option in the portfolio.

If the portfolio was valued using a parameterized vol surface – I.e. if there was a functional relationship defined for the volatility surface – then it would be relatively easy to calculate the new volatility surface and find the change in volatility for each option.

e.g. if the book was marked to a simple linear equation Put Vol = 40% + 0.7% * (put delta + 0.50)/5

Then a 25Delta put would be marked at 43.5%

If the skew-vega is calculated using +1vol rise in 25delta put vol, then one could easily recalculate the new function to be PutVol = 40% + 0.8% * (put delta +0.5)/5

And thus calculate the vol for any delta option and then apply the change in vol to the vega to get the total Skew vega.

But if the portfolio is not marked to a defined curve function, but is marked to the IV's of the market or marked to trader provided independent vols - the calculation of individual option shifts become quite challenging. This is quite common in managing the risk of a listed options portfolio, where all options and greeks are calculated based on market implied vols. To solve this problem, we have to use some creative mathematical techniques.

Rotating the curve

When a vol surface is moved by a certain OTM point but pivoted at the ATM point (as it is held constant), effectively the curve is being rotated. Thus regardless of the shape or functional form of the curve, the entire curve is experiencing a rotation, e.g.

25d
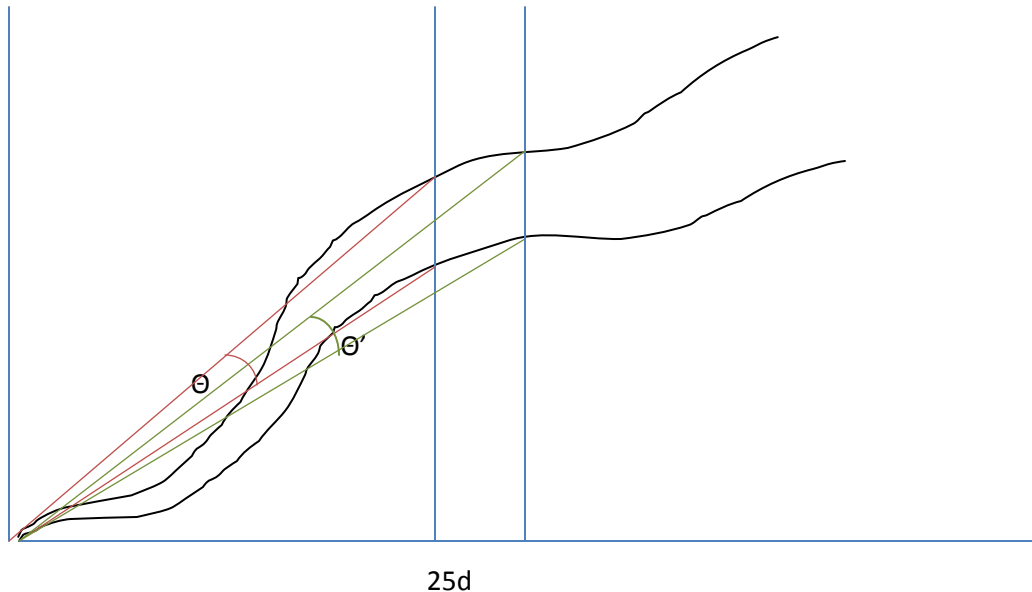
Above we show a random vol surface, which has been rotated due to a change in the vol at say the 25d point. The rotation is represented by angle Θ

At some other delta point D', there has been a shift in vol as well. It can be shown that the shift in vol at D' projects an angle Θ' which is approximately equal to Θ

Thus a change in the vol at a particular delta will shift the vols at all other delta points by different amounts, but the rotation of each point will be the same.

Thus if we can translate the Vol shift into a angle of rotation and vice versa, we can transform a vol shift at one point of the curve into a vol shift at any other point of the curve – *without any knowledge of the shape or functional form of the curve!*

The model

Assume the Vol at Delta d, is the vol we are shifting, which is V_d

And the ATM vol is V_0

Let $V = V_d - V_0$

Lets D = difference in delta from ATM , thus D = 50 – Abs(d)

**Note since puts are negative delta and we want to treat ITM Calls (Puts) as OTM Puts (calls) , then we use D = Abs(50-Abs(d))

From basic trigonometry we thus have:

$$\theta_d = \tan^{-1}(V/D)$$

Note that the $\tan^{-1}$ function is also called Arctan

Assume the vol at delta d is being moved by ΔV

Thus the new angle at the shocked vol will be

$$\theta'_d = \tan^{-1}\left(V + \frac{\Delta V}{D}\right)$$

And thus the change in angle due to the shift in the vol at delta d will be:

$$\theta = \theta'_d - \theta_d$$

$$\theta = \tan^{-1}\left(V + \frac{\Delta V}{D}\right) - \tan^{-1}(V/D) \qquad \text{............ (2)}$$

Now assume we want to figure out the vol change at some other delta point, D' (again this is relative to the ATM, so a 10 delta put would mean D' = 40%).

Say V' is the current IV difference from the ATM vol for D',

The current angle at D' would be

$$\theta_{d'} = \tan^{-1}\left(\frac{V'}{D'}\right)$$

The rotated angle would be

$$\tan^{-1}\left(\frac{V'}{D'}\right) + \theta$$

And thus the change in vol at D' would be:

$$\Delta V' = D'.\tan\left[\tan^{-1}\left(\frac{V'}{D'}\right) + \theta\right] - V' \qquad \text{............ (3)}$$

Formulas (2) and (3) form the model, whereby we first figure out rotation due to change in vol and then apply this rotation to any other delta option.

Once we calculate the vol changes for each option, we just multiply this by their vega to get the overall vega due to the rotation.

If the Skew-vega is defined as a move in both call and put vol curves, then we do the same exercise independently for the calls (along with ITM puts) and puts (along with ITM Calls)